Smoothed Particle Hydrodynamics: a Lagrangian Approach to Hydraulics

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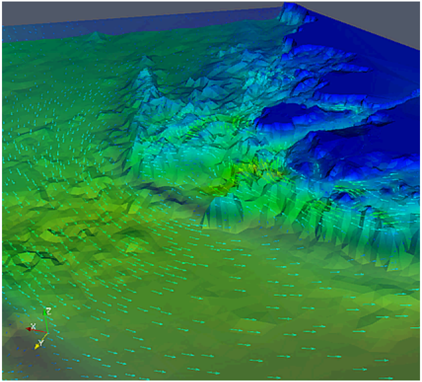
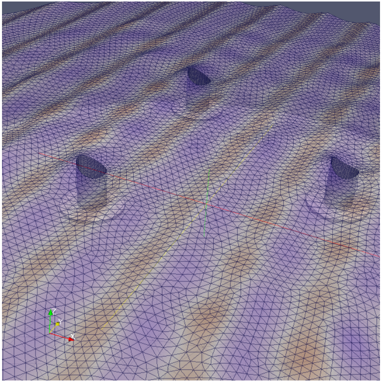
Introduction: CFD for free-surface flows

- **Traditional CFD can help you in many applications ...**

Finite Elements



Boundary Elements



http://www.opentelemac.org

Dombre *et al.* (2015), J. Ocean Engng. Marine Energy **1**(1):55–76

- ... **but about these flows?**

Particle methods

- **Why particle methods are useful**

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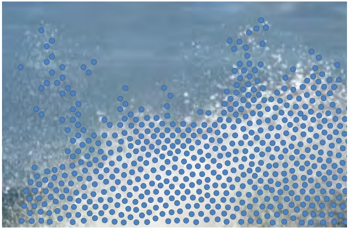

Particle methods

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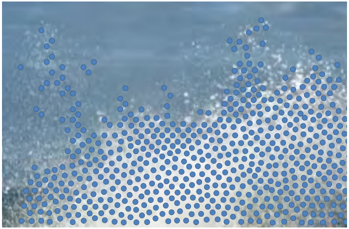

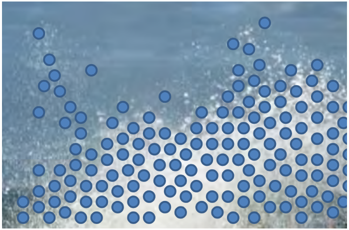

Particle methods

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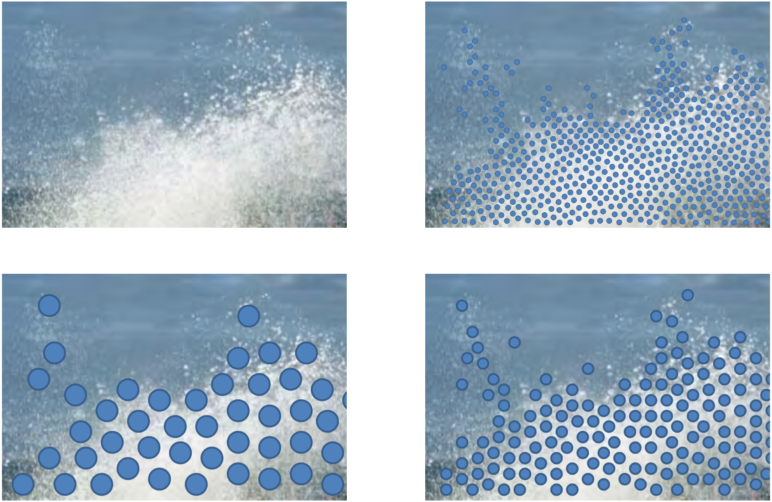

Smoothed
Particle
Hydrodynamics

What is a 'particle'?

- **Going to Wikipedia**
  - 'A particle is a minute fragment or quantity of matter'
- **Usual meanings in science**
  - Smallest constituents of matter (Standard Model)
  - Nanoparticles, colloidal particles
  - Dust, powder, ashes
  - Sediment grains, water droplets
  - etc.

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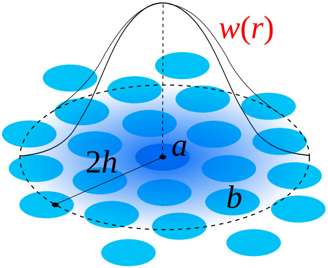
- **The duality of 'particles' in SPH**
  - They are material points
  - They have volume, mass, pressure, density, etc.

1. Fundamentals of SPH

SPH Particles and kernel

- Particle a has position $\mathbf{r}_a$, mass m_a, volume V_a, etc.
- Particle interaction are computed using the '**kernel**' $w(r)$
- The support of w has size $2h$, $h =$ **smoothing length**
- w is normalised:

$$\int_\Omega w(r) d\mathbf{r} = 1$$



Lucy, L.B. (1977), Astron. J. **82**:1013–1024

Gingold, R.A., Monaghan, J.J. (1977), Mon. Not. R. Astron. Soc. **181**:375–389

SPH interpolation

- **Continuous** interpolation:

$$
\begin{aligned}
f(\mathbf{r}) &= \int_\Omega f(\mathbf{r}')\delta(\mathbf{r}-\mathbf{r}')d\mathbf{r}' \\
&= \int_\Omega f(\mathbf{r}')w(|\mathbf{r}-\mathbf{r}'|)d\mathbf{r}' + \mathcal{O}\left(h^2\right)
\end{aligned}
$$

SPH interpolation

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- **Discrete** interpolation:

$$
f(\mathbf{r}_a) \equiv f_a \approx \sum_b V_b f_b w(|\mathbf{r}_a - \mathbf{r}_b|)
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SPH interpolation

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- **Discrete** interpolation:

$$f(\mathbf{r}_a) \equiv f_a \approx \sum_b V_b f_b w(|\mathbf{r}_a - \mathbf{r}_b|)$$

- Case of the **density** $f \equiv \rho = m/V$:

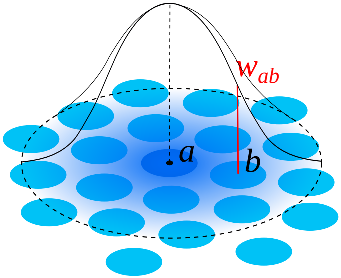
$$\boxed{\rho_a = \sum_b m_b w_{ab}}$$

  - $r_{ab} \doteqdot |\mathbf{r}_a - \mathbf{r}_b|$
  - $w_{ab} \doteqdot w(r_{ab})$

SPH interpolation

- **Illustration**

$$\rho_a = \sum_b m_b w_{ab}$$



- **Illustration**

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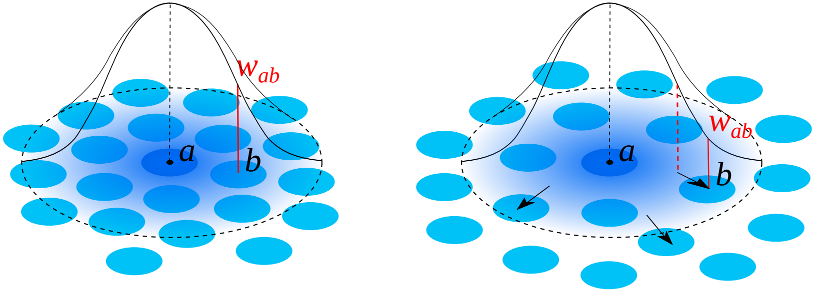

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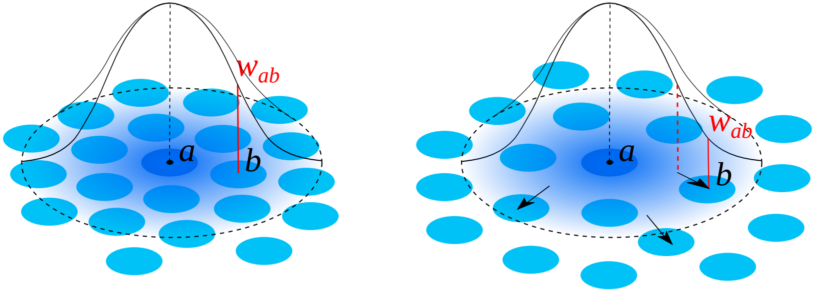

larger ρ_a

smaller ρ_a

$$\dot{\rho}_a = -\rho_a \left(\nabla \cdot \mathbf{v}\right)_a < 0$$

SPH divergence

- **Differentiating** the density:

$$\begin{aligned} \mathrm{d}\rho_a &= \sum_b m_b \mathrm{d}w_{ab} \\ &= \sum_b m_b \left(\mathrm{d}\mathbf{r}_a - \mathrm{d}\mathbf{r}_b\right) \cdot \nabla w_{ab} \end{aligned}$$

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- Dividing by $\mathrm{d}t$ (with $\mathrm{d}\mathbf{r}_a = \mathbf{v}_a \mathrm{d}t$):

$$\dot{\rho}_a = \underbrace{\sum_b m_b \left(\mathbf{v}_a - \mathbf{v}_b\right) \cdot \nabla w_{ab}}_{\approx -\rho_a (\nabla \cdot \mathbf{v})_a}$$

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- SPH **divergence operator**:

$$\boxed{\mathsf{D}_a \left\{\mathbf{v}_b\right\} \doteqdot -\frac{1}{\rho_a} \sum_b m_b \left(\mathbf{v}_a - \mathbf{v}_b\right) \cdot \nabla w_{ab} \approx \left(\nabla \cdot \mathbf{v}\right)_a}$$

SPH gradient

- On similar grounds, we get the SPH **gradient operator**:

$$\boxed{\mathbf{G}_a\left\{p_b\right\} \doteqdot \rho_a \sum_b m_b \left(\frac{p_a}{\rho_a^2} + \frac{p_b}{\rho_b^2}\right) \nabla w_{ab} \approx \left(\nabla p\right)_a}$$

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- One can define **inner products**:

$$\begin{aligned}
\langle\{f_a\},\{g_a\}\rangle &\doteq \sum_a V_a f_a g_a \approx \int_\Omega f\left(\mathbf{r}\right) g\left(\mathbf{r}\right) d\mathbf{r} \\
\langle\{\mathbf{f}_a\},\{\mathbf{g}_a\}\rangle &\doteq \sum_a V_a \mathbf{f}_a \cdot \mathbf{g}_a \approx \int_\Omega \mathbf{f}\left(\mathbf{r}\right) \cdot \mathbf{g}\left(\mathbf{r}\right) d\mathbf{r}
\end{aligned}$$

- Key property: **G** et D are **skew-adjoint**:

$$\langle \mathbf{G}_a\{p_b\},\{\mathbf{v}_a\}\rangle = -\langle\{p_a\},\mathrm{D}_a\{\mathbf{v}_b\}\rangle$$

Summary of SPH operators

- SPH **divergence**:

$$\mathsf{D}_a\left\{\mathbf{v}_b\right\} \doteqdot -\frac{1}{\rho_a}\sum_b m_b\left(\mathbf{v}_a-\mathbf{v}_b\right)\cdot\nabla w_{ab} \approx \left(\nabla\cdot\mathbf{v}\right)_a$$

- SPH **gradient**:

$$\mathbf{G}_a\left\{p_b\right\} \doteqdot \rho_a\sum_b m_b\left(\frac{p_a}{\rho_a^2}+\frac{p_b}{\rho_b^2}\right)\nabla w_{ab} \approx \left(\nabla p\right)_a$$

- SPH **Laplacian**:

$$\mathbf{L}_a\left\{\mathbf{v}_b\right\} \doteqdot 2\sum_b V_b\left(\mathbf{v}_a-\mathbf{v}_b\right)\frac{\mathbf{r}_{ab}}{r_{ab}^2}\cdot\nabla w_{ab} \approx \left(\nabla^2\mathbf{v}\right)_a$$

  - $\mathbf{r}_{ab} \doteqdot \mathbf{r}_a-\mathbf{r}_b$
  - $r_{ab} \doteqdot \left|\mathbf{r}_{ab}\right|$

Standard Weakly Compressible SPH model (WCSPH)

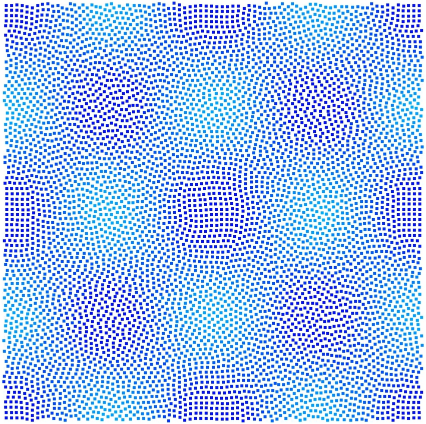
- SPH form of the Lagrangian **Navier-Stokes** equations:

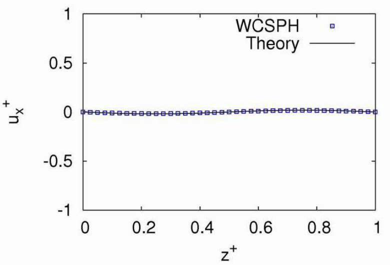
$$
\begin{aligned}
\dot{\mathbf{v}}_a &= -\frac{1}{\rho_a}\mathsf{G}_a\{p_b\} + \frac{\mu}{\rho_a}\mathsf{L}_a\{\mathbf{v}_b\} + \mathbf{g} \\
\dot{\mathbf{r}}_a &= \mathbf{v}_a \\
\dot{\rho}_a &= -\rho_a \mathsf{D}_a\{\mathbf{v}_b\} \\
p_a &= \frac{\rho_0 c_0^2}{\gamma}\left(\frac{\rho_a^\gamma}{\rho_0^\gamma} - 1\right)
\end{aligned}
$$

- **Definitions**:
  - $\mathbf{v}_a$, ρ_a, p_a, $\mathbf{r}_a$: particle velocity, density, pressure, position
  - $\mu \doteqdot \rho_0 \nu$: fluid dynamic viscosity
  - ρ_0: reference fluid density
  - $c_0 \doteqdot 10U_{max}$: numerical speed of sound
  - $\gamma = 7$ for water

Monaghan, J.J. (1994), J. Comput. Phys. **110**:399–406

Example: Taylor-Green vortices





Energy

- Behaviour of the **macroscopic energy** (Hamiltonian):

$$H = \sum_a m_a \left(\frac{1}{2}\mathbf{v}_a^2 + e_{int,a} - \mathbf{g}\cdot\mathbf{r}_a \right) \qquad \dot{e}_{int} = \frac{\partial e_{int}}{\partial \rho}\dot{\rho} = \frac{p}{\rho^2}\dot{\rho}$$

$$\begin{aligned}
\dot{H} &= \sum_a m_a \left[(\dot{\mathbf{v}}_a - \mathbf{g}) \cdot \mathbf{v}_a + \frac{p_a}{\rho_a^2}\dot{\rho}_a \right] \\
&= -\sum_a V_a \left(\mathsf{G}_a \{p_b\} \cdot \mathbf{v}_a + p_a \mathsf{D}_a \{\mathbf{v}_b\} \right) \quad +\text{viscosity} \\
&= -\langle \mathsf{G}_a \{p_b\}, \{\mathbf{v}_a\} \rangle - \langle \{p_a\}, \mathsf{D}_a \{\mathbf{v}_b\} \rangle \quad +\text{viscosity} \\
&= \qquad \text{viscosity}
\end{aligned}$$

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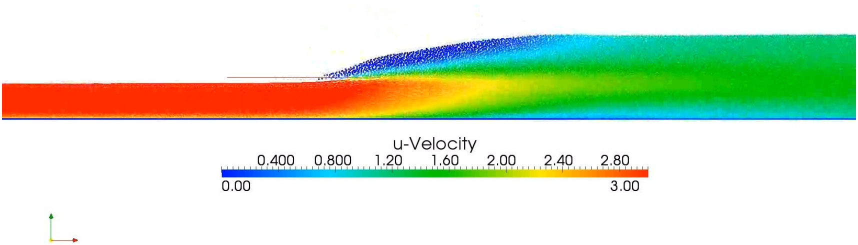
\end{aligned}$$

- This is similar to...

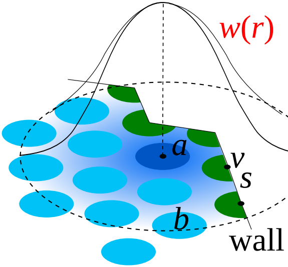
$$\dot{H} = -\int_\Omega \left(\mathbf{v} \cdot \nabla p + p \nabla \cdot \mathbf{v} \right) d\mathbf{r} = \int_{\partial\Omega} p \mathbf{v} \cdot \mathbf{n} \, d\Gamma + \text{viscosity}$$

Hydraulic jump



Modelling walls in SPH

- Solid walls can be discretised with **boundary elements** s and truncated particles called **vertex particles** v



Integral of w over the kernel support:

$$\gamma_a \doteqdot \int_\Omega w\left(|\mathbf{r}_a - \mathbf{r}|\right) d\mathbf{r} \leq 1$$

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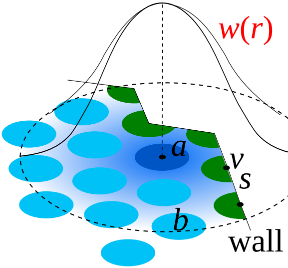

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$$\begin{aligned}
\rho_a &= \frac{1}{\gamma_a} \sum_b m_b w_{ab} \\
\dot{\rho}_a &= \frac{1}{\gamma_a} \sum_b m_b \left(\mathbf{v}_a - \mathbf{v}_b\right) \cdot \nabla w_{ab} - \frac{\dot{\gamma}_a}{\gamma_a^2} \sum_b m_b w_{ab}
\end{aligned}$$

Modelling walls in SPH

- SPH divergence **with boundary terms**:

$$\mathrm{D}_a^{\gamma}\left\{\mathbf{v}_b\right\} \doteqdot -\frac{1}{\gamma_a \rho_a} \underbrace{\sum_b m_b \left(\mathbf{v}_a - \mathbf{v}_b\right) \cdot \nabla w_{ab}}_{\text{contribution of particles}} + \frac{1}{\gamma_a} \underbrace{\sum_s \left(\mathbf{v}_a - \mathbf{v}_s\right) \cdot \nabla \gamma_{as}}_{\text{contribution of boundary elements}}$$

$$\text{where} \quad \nabla \gamma_{as} \doteqdot \int_s w\left(\left|\mathbf{r}_a - \mathbf{r}\right|\right) \mathbf{n}_s \, d\Gamma$$

Ferrand *et al.* (2012), Int. J. Num. Meth. Fluids **71**(4):446–472

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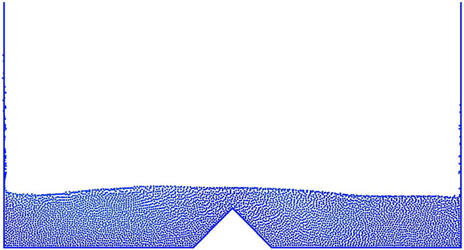
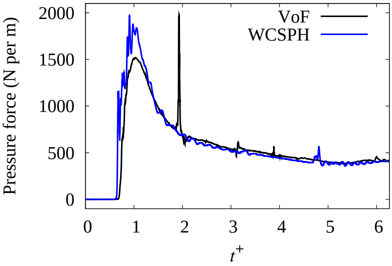
$$\text{where} \quad \nabla \gamma_{as} \doteqdot \int_s w\left(\left|\mathbf{r}_a - \mathbf{r}\right|\right) \mathbf{n}_s \, d\Gamma$$

- γ_a and $\nabla \gamma_{as}$ can be computed analytically
- The velocities of boundary elements $\mathbf{v}_s$ are prescribed
- Similar treatments are applied to $\mathbf{G}_a^{\gamma}\left\{p_b\right\}$ and $\mathbf{L}_a^{\gamma}\left\{\mathbf{v}_b\right\}$

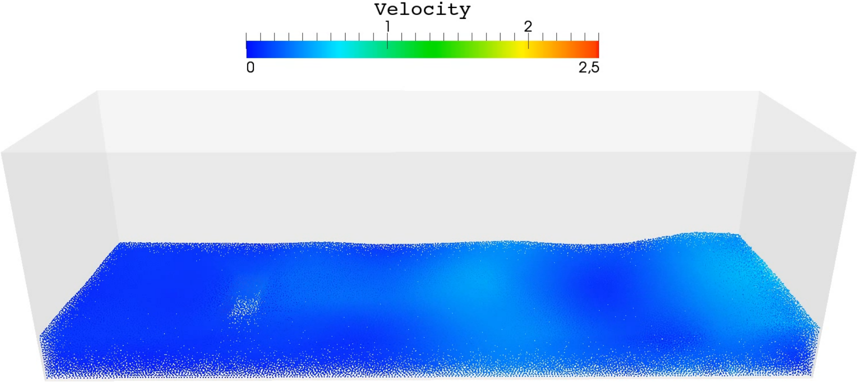
Ferrand *et al.* (2012), Int. J. Num. Meth. Fluids **71**(4):446–472

Water collapse on a wedge



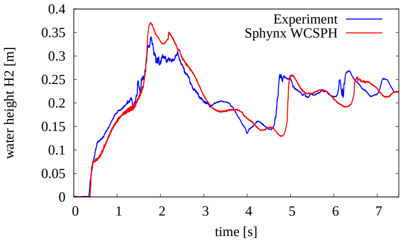


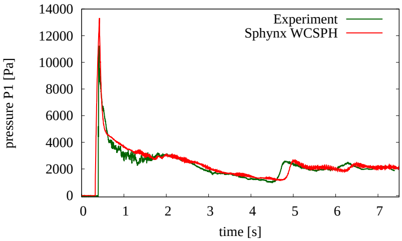
3D Water collapse

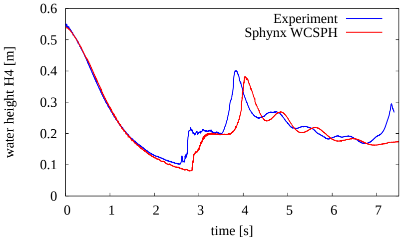


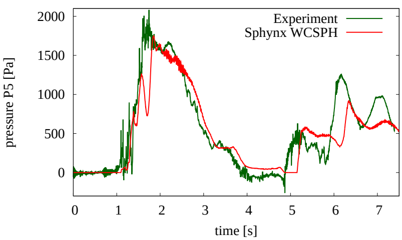
Validation

- Comparaison against experiments (Kleefsman *et al.*, 2005)









Kleefsman *et al.* (2005), J. Comput. Phys. **206**:363–393

2. Application to hydraulics

Application fields

- **SPH can be used to solve**:
  - (Reynolds-Averaged) Navier–Stokes equations
  - Pressure Poisson equation (ISPH): $\mathsf{L}_a^{\gamma}\{p_b\} = \frac{\rho}{\Delta t}\mathsf{D}_a^{\gamma}\{\mathbf{v}_b\}$
  - Turbulent $k-\epsilon$ model: $\dot{k}_a = P_a + \mathsf{L}_a^{\gamma}\{\nu_{T,b}, k_b\} - \epsilon_a$
  - Temperature equation: $\dot{T}_a = K\mathsf{L}_a^{\gamma}\{T_b\}$
  - Shallow water equations: $\dot{\mathbf{U}}_a = -g\mathbf{G}^{\gamma}\{\eta_a\}$

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  - Surface tension
  - Sediment transport
  - Porous media
  - Rigid bodies in fluids
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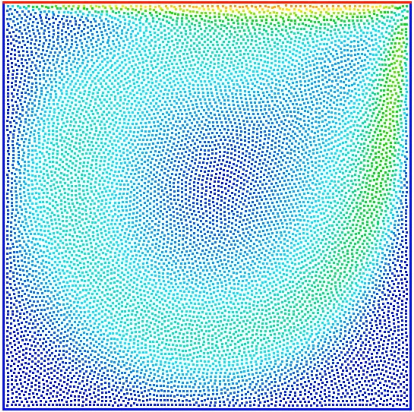
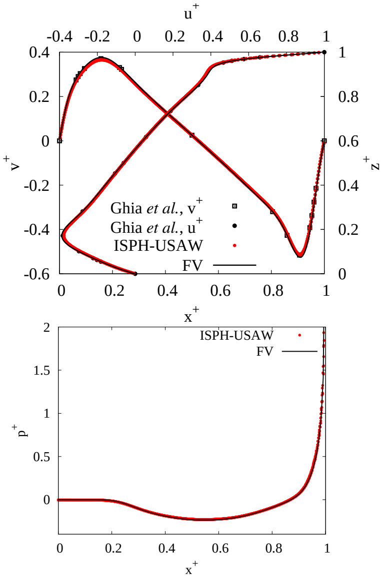
- **... but is SPH as good as mesh methods**?

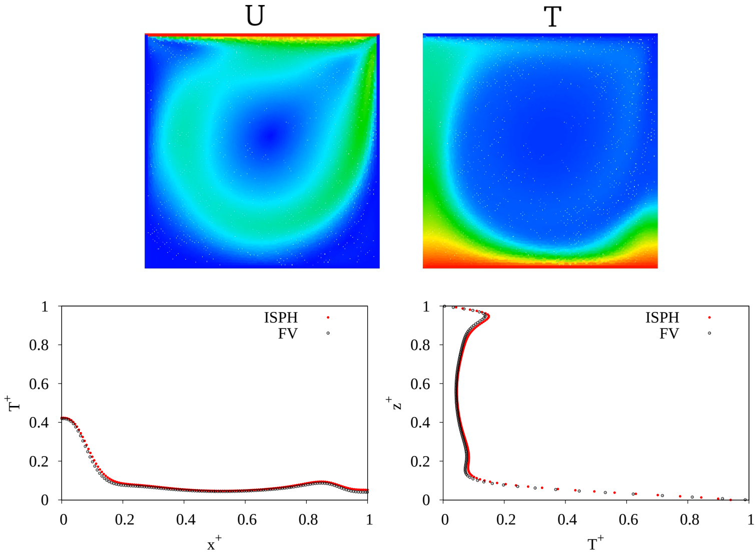
Lid-driven cavity flow





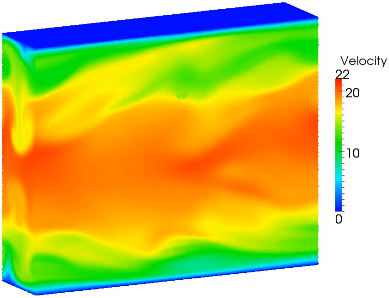
Leroy *et al.* (2014), J. Comput. Phys. **261**:106–129

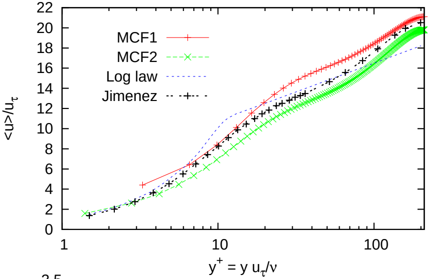
Heated lid-driven cavity flow

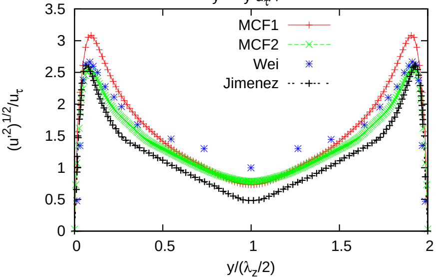


Leroy *et al.* (2015), Int. J. Num. Meth. Fluids DOI: 10.1002/fld.4025

Channel DNS







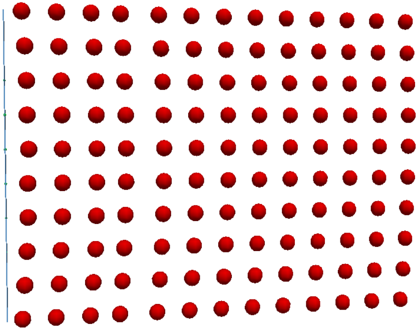
- $Re_\tau = 210$
- ca. 10 million particles

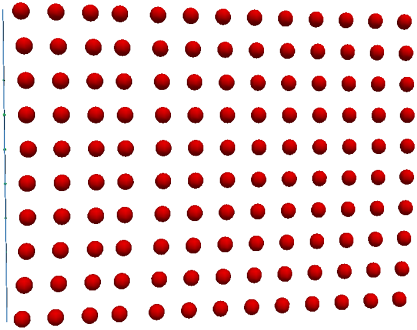
\+ + SPH

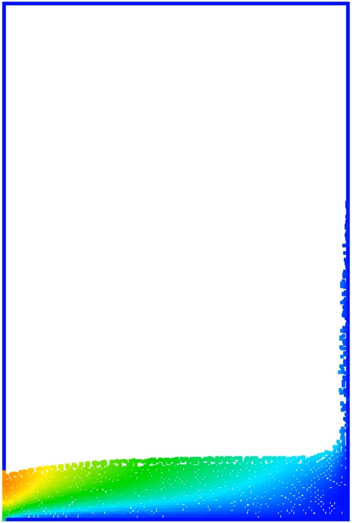
\+ * Mesh methods

Mayrhofer *et al.* (2015), Comput. & Fluids **115**:86–97

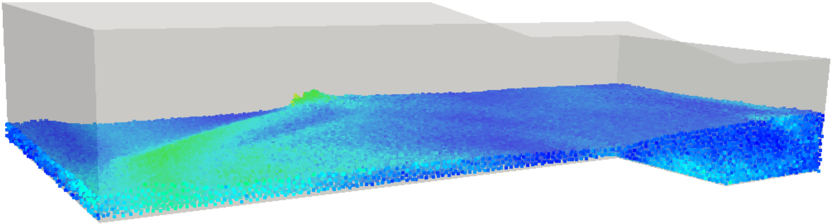
Open boundaries in SPH



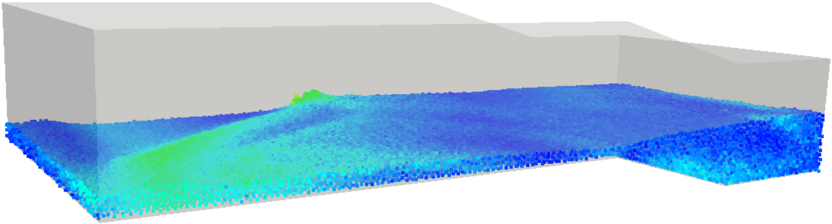


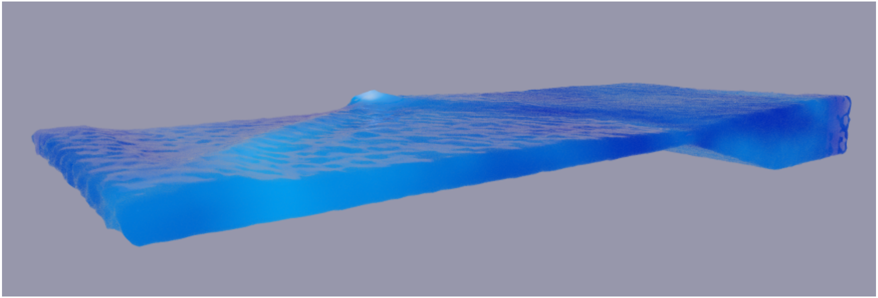


Open boundaries in SPH



Open boundaries in SPH





Oil spill containment boom

www.cedre.fr/Cedre

Violeau *et al.* (2007), Coastal Eng. **54**:895–913

Oil spill containment boom

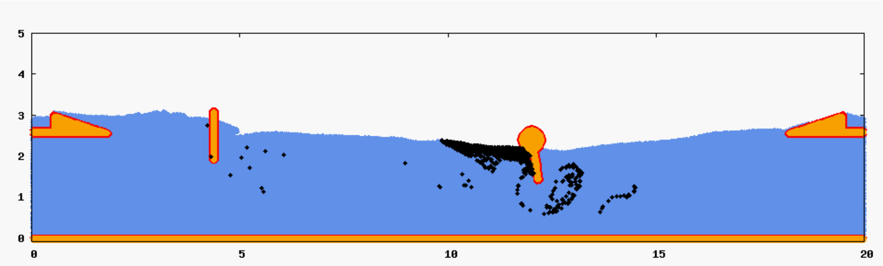
- Oil spill **stability criterion**:

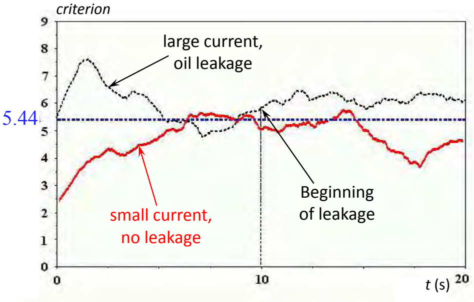
$$\frac{\text{buoyancy}}{\text{TKE production}} \sim \frac{\rho_{oil} S \sqrt{k}}{(\rho - \rho_{oil})\, g} \leq \sqrt{\frac{8}{3C_\mu}} \approx 5.44$$

www.cedre.fr/Cedre

Violeau *et al.* (2007), Coastal Eng. **54**:895–913

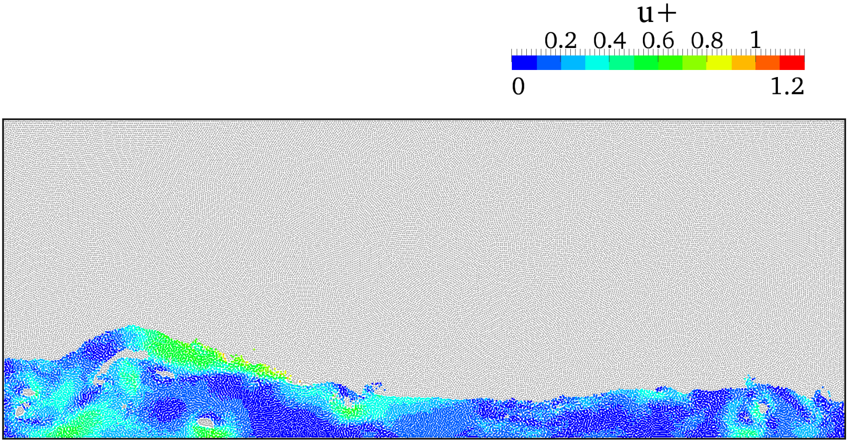
Oil spill containment boom





Multi-phase SPH

- **Example**: water and air
  - Requires modified operators $\mathbf{G}^{\gamma}$, D_a^{γ} and L_a^{γ}



High-Performance Computing for SPH

- Each particule interacts with about 250 neighbours en 3D(!)
- SPH is thus time consuming...

Hérault *et al.* (2010), J. Hydr. Res. **48**:74–79

High-Performance Computing for SPH

- Each particule interacts with about 250 neighbours en 3D(!)
- SPH is thus time consuming...
- Solutions:
  - **Massive parallelism** on CPU clusters
  - **Graphic cards** (GPU) computing (Hérault *et al.*, 2010)

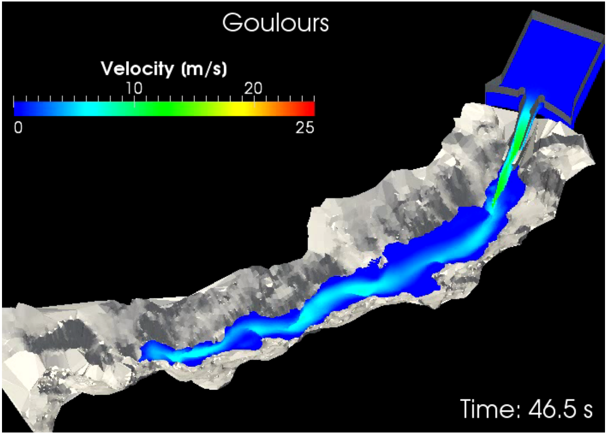
Bull Tera 100
100 M€

NVidia GeForce GTX 275
1800€

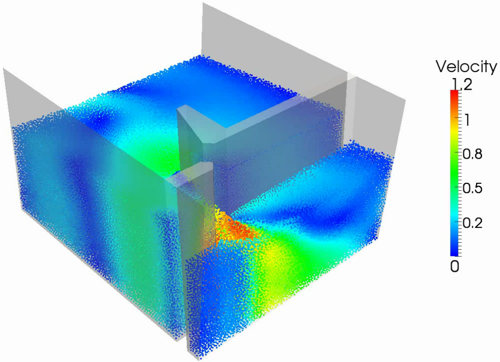
Hérault *et al.* (2010), J. Hydr. Res. **48**:74–79

HPC with massive clusters

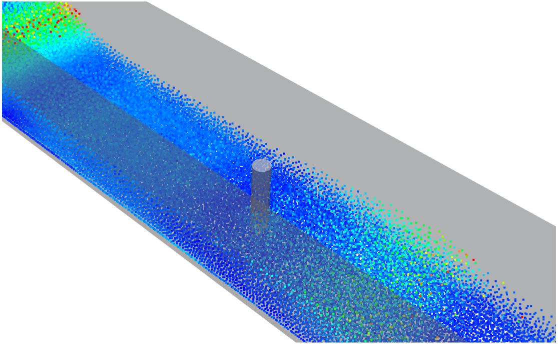
- Ski-jump **dam spillway**
  - ca. 1.3 million particles, 1.2 million boundary elements
  - 2 days CPU on 512 nodes (IBM BlueGene) for 93,000 iterations

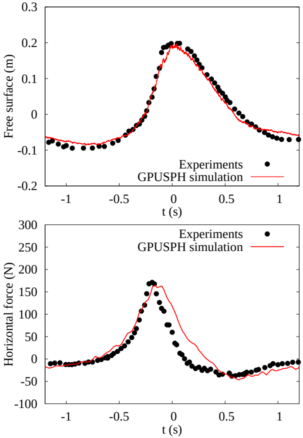


- Vertical slot **fish pass**
  - **GPUSPH** open-source code (http://www.gpusph.org)
  - ca. 370,000 particles, 80,000 boundary elements
  - 5 hours on a single graphic card for 150,000 iterations

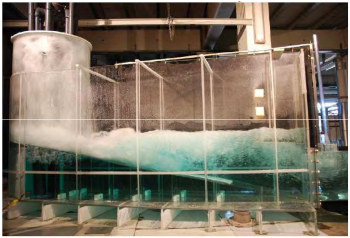


- **Offshore wind turbine**
  - ca. 560,000 particles, 200,000 boundary elements
  - 8 hours on 2 graphic cards for 300,000 iterations



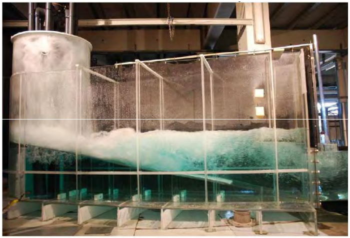


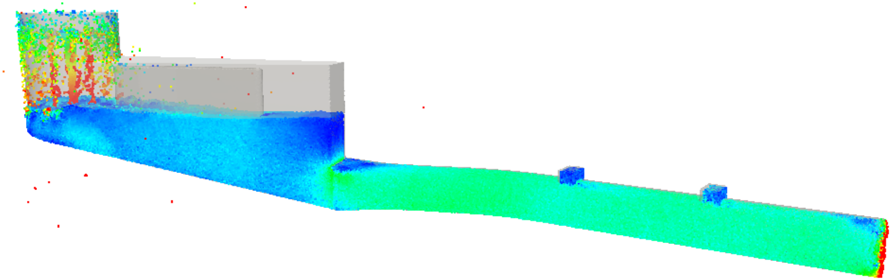
- **Hydro-restoration tank**
  - Mixed confined/free-surface flow



On-going projects

- **Hydro-restoration tank**
  - Mixed confined/free-surface flow





On-going projects

- Ogee-type **dam spillway**

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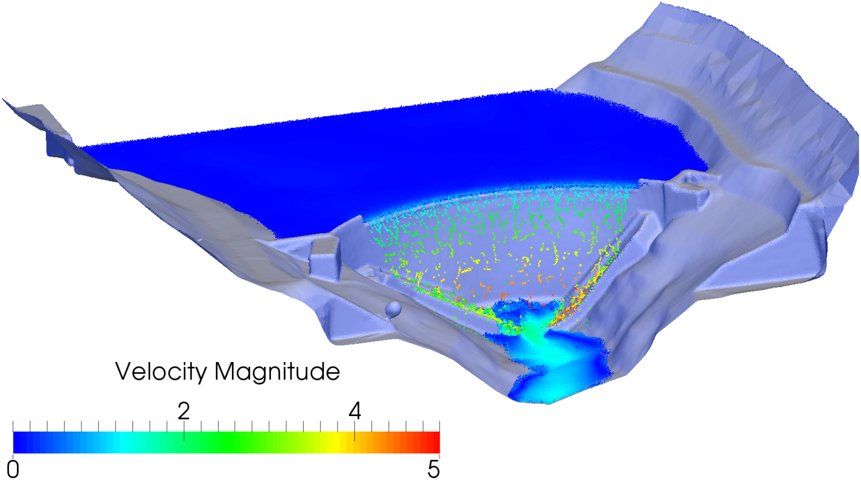
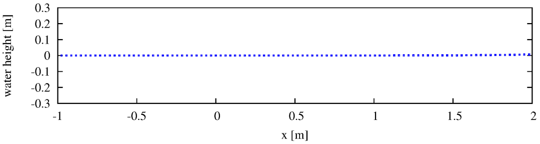
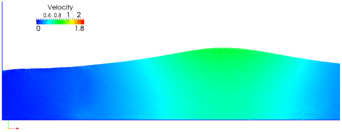
- Ogee-type **dam spillway**



Lagrangian–Eulerian coupling

- **The future of SPH?**
  - Fluid (SPH) – Structure (mesh)
  - Fluid (SPH) – Fluid (mesh)





Lagrangian–Eulerian coupling

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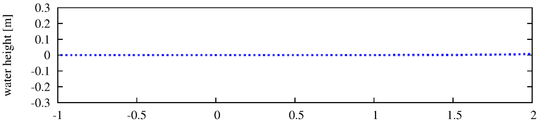

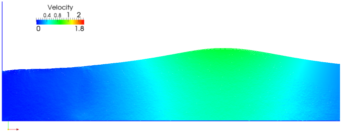

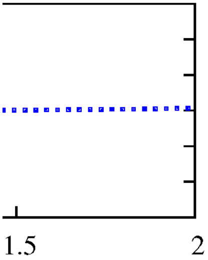

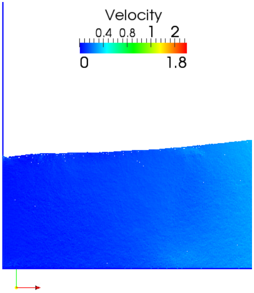

3. Conclusions

More about SPH

- **SPHERIC**: **SPH E**uropean **R**esearch **I**nterest **C**ommunity
  - Created October 2005
  - Hosted by ERCOFTAC
  - 70 member institutes worldwide
  - Annual conference (ca. 100 attendees)
  - Biannual newsletter
  - **G**rand **C**hallenge **W**orking **G**roup

  http://wiki.manchester.ac.uk/spheric

More about SPH

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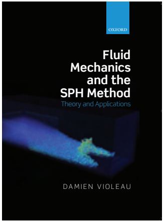
- **Further reading** (Violeau, 2012)
  - Understanding the fundamentals of fluids
  - Introducing SPH for fluids
  - 616 pages, 111 figures
  - Published by Oxford University Press

  http://ukcatalogue.oup.com/product/9780199655526.do



Violeau, D. (2012), *Fluid Mechanics and the SPH Method*, Oxford Univ. Press

Hall of fame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Antoine Joly
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Researcher Engineer


Arno Mayrhofer
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PhD student


Louise Fratter
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- V. Hergault, ALTEN (France)

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