Dealing with complex transport problems in groundwater: when one is expected to provide exact solutions with little to none information

Xavier Sanchez-Vila
Universitat Politècnica de Catalunya - BarcelonaTech
“Hydraulic engineering was at the forefront of science for centuries. The end of the 20th century marked a change of perception in our society, especially in developed countries, with a focus on environmental sustainability and management.” Hubert CHANSON, 13th Ippen Award Lecture, 2003
Engineers are **problem solvers**. We are expected to come up with solutions. For this purpose we have or ask for information.

We ALWAYS claim we need more information, but here I want to highlight some significant differences between surface and subsurface hydrology.
First, Surface hydrologists consider a nice water cycle where they include runoff, evapotranspiration and something called losses.

We hydrogeologists have a hard time trying to tell students that you do not lose these losses, but rather they are our input and we call them AQUIFER RECHARGE (eventually you can drink it!)
Second,

You do see things!

… and then act accordingly
We just do not see anything … so just guessing?
Third,

You can control things

A tracer test in a river. You place a dye and watch it as it moves. You can take pictures!
Hydraulic engineering for centuries: Is hydro-geo engineer the oldest job in history?

- The qanat technology is known to have developed in middle-East 3000 years ago.
- Iran has the oldest qanat (circa 1000 BC) and longest (71 km). From there extensions to several locations in middle-east
- Also, a qanat-like system called the Turpan Water System originated in China during the Han Dynasty (around 206 BC).
Change of perception: Hitting the news?

Media most likely would cover a new dam put in service than placing a pump in a borehole to get some water.

But some problems in hydrogeology are really affecting millions of people:
- in terms of quantity: refugee camps
- in terms of quality: Bangladesh
In Bangladesh nearly 90% of the population uses groundwater as its primary source of fresh water.

Up to 77 million people in Bangladesh have been exposed to toxic levels of arsenic (naturally occurring) from drinking water.

WHO said the exposure was "the largest mass poisoning of a population in history".

Death counting is in the tens of thousands.
We DO face complex/difficult problems

- Early XXth Century: Flow in porous media/well hydraulics. Emphasis: QUANTITY
- Second part of the century: Geochemistry, transport of conservative solutes: Emphasis: QUALITY
- 80’s on: Stochastic hydrogeology, modelling
- New stuff every decade: surface/subsurface interactions, coastal aquifer dynamics, geothermal, vadose zone infiltration,…
- multispecies reactive transport, CO2 sequestration, climate change & alternative resources related to integrated water management, risk evaluation, …
And WE CANNOT SEE ANYTHING! So, treating geological complexity

- We need kind of geological reconstruction

- Very similar to CSI

- With little (to no) information, reconstruct as best as possible the undersampled formation
Original figure. Selection of 10 random samples
Classical Kernel Regression
Steering Kernel Regression Second Order (after 2 iterations)
Complexity in processes adds to that in geology; e.g., multispecies reactive transport

- Goal: Providing “simple”, understandable solutions for evaluating the fate of reacting plumes (complex multispecies reactive transport problems).

- Is it really possible?
YES!!!!
The idea of pollution

Pollution starts at one point and moves. Along the movement several processes occur to change direction and to spread the problem so that the neighbours are "not-happy"
The idea of groundwater pollution

- You do not see anything until it is too late, PERFECT FOR LAW SUES
- Famous cases:
  - Hinkley, CA: Cr (VI)
  - Woburn, MA: Trichloroethylene

![Chemical structure of Cl₃C=CCl₃](image)
Fortunately not everything GW carries is a pollutant. Still GW solute transport can be relevant in other biogeochemical problems.

**ARTIFICIAL RECHARGE** (different waters mix, and then react)
CAVE FORMATION
(limestone dissolution and reprecipitation)
Mixing of two (or more) waters in perfect geochemical equilibrium (regarding aqueous species, biological species and minerals) produces local disequilibrium. Reactions will then take place to re-equilibrate the system (acid/base, redox, precip./dissol., adsorption/desorption, biological growth).

**Example not so rare: 19 chemical species and 13 reactions THIS MEANS SOLVING A SYSTEM OF 32 NON-LINEAR PDE’s**

Coupled flow and reactive transport problem is modeled with numerical codes.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>log $K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{CaCl}^+ = \text{Ca}^{2+} + \text{Cl}^-$</td>
<td>0.6938</td>
</tr>
<tr>
<td>$\text{CaCl}_2(\text{aq}) = \text{Ca}^{2+} + 2\text{Cl}^-$</td>
<td>0.6283</td>
</tr>
<tr>
<td>$\text{CaHCO}_3^+ = \text{Ca}^{2+} + \text{HCO}_3^-$</td>
<td>-1.0606</td>
</tr>
<tr>
<td>$\text{CaOH}^+ + \text{H}^+ = \text{Ca}^{2+} + \text{H}_2\text{O}$</td>
<td>12.9321</td>
</tr>
<tr>
<td>$\text{CO}_2(\text{aq}) + \text{H}_2\text{O} = \text{HCO}_3^- + \text{H}^+$</td>
<td>-6.3636</td>
</tr>
<tr>
<td>$\text{CO}_3^{2-} + \text{H}^+ = \text{HCO}_3^-$</td>
<td>10.3524</td>
</tr>
<tr>
<td>$\text{OH}^- + \text{H}^+ = \text{H}_2\text{O}$</td>
<td>14.0707</td>
</tr>
<tr>
<td>$\text{HCl}(\text{aq}) = \text{H}^+ + \text{Cl}^-$</td>
<td>0.6693</td>
</tr>
<tr>
<td>$\text{NaCl}(\text{aq}) = \text{Na}^+ + \text{Cl}^-$</td>
<td>0.7811</td>
</tr>
<tr>
<td>$\text{NaCO}_3^- + \text{H}^+ = \text{Na}^+ + \text{HCO}_3^-$</td>
<td>9.8145</td>
</tr>
<tr>
<td>$\text{NaHCO}_3(\text{aq}) = \text{Na}^+ + \text{HCO}_3^-$</td>
<td>-0.1715</td>
</tr>
<tr>
<td>$\text{NaOH}(\text{aq}) + \text{H}^+ = \text{Na}^+ + \text{H}_2\text{O}$</td>
<td>14.2479</td>
</tr>
<tr>
<td>$\text{CaCO}_3(\text{s}) + \text{H}^+ = \text{Ca}^{2+} + \text{HCO}_3^-$</td>
<td>1.8789</td>
</tr>
</tbody>
</table>
Reactions

- Best place to occur: reactors
The Concept of Mixing

Mixing without reactors?

Confluence Amazon – Rio Negro
THE CONCEPT OF MIXING

Mixed, not stirred! (Connery, 1964)
FIRST, WE GO LOCAL

Reactions driven by mixing

Alternatives: reactions in permanent equilibrium (instantaneous eq) or not. Depends on the Damköhler number, i.e. ratio of two characteristic times

$$Da = \frac{\tau_D}{\tau_r}$$

Equilibrium means $$Da \to \infty$$ and trajectory is the grey line (and arrow)

$$Da \to 0$$ means slow mixing (in relative terms), and r follows red arrows.

Reality could be something in between

(a) $$c_1 c_2 = K$$
WHAT IF NON-EQUILIBRIUM?
MAPPING REACTIONS

Not an easy task indeed!

So a new methodology has recently been proposed.
Assume 2 species (e.g. $\text{SO}_4^{2-}$ and $\text{Ca}^{2+}$) in eq. with gypsum

Step 1: Chemical system

**Reaction**

$$\text{Ca}^{2+} + \text{SO}_4^{2-} \iff \text{CaSO}_4 \Rightarrow \left[ \text{Ca}^{2+} \right] \cdot \left[ \text{SO}_4^{2-} \right] = K$$

**Components:**

$$u = \left[ \text{Ca}^{2+} \right] - \left[ \text{SO}_4^{2-} \right] \quad \text{is conservative!}$$

Step 2: Solve transport of $u$

(1) $$\frac{\partial (\phi C_1)}{\partial t} + \mathbf{q} \cdot \nabla C_1 - \nabla \cdot (\mathbf{D} \nabla C_1) = -r$$

(2) $$\frac{\partial (\phi C_2)}{\partial t} - \mathbf{q} \cdot \nabla C_2 + \nabla \cdot (\mathbf{D} \nabla C_2) = -r$$

(1)-(2) yields:

$$\frac{\partial (\phi u)}{\partial t} = L_t (u) \quad \text{CONSERVATIVE!!!}$$
Analytical solution for 2 species

Step 3: Speciation

Solve

\[ u = C_1 - C_2 \]

Together with

\[ C_1 \cdot C_2 = K \]

\[ C_1 = \frac{u + \sqrt{u^2 + 4K}}{2} \]

\[ C_2 = \frac{-u + \sqrt{u^2 + 4K}}{2} = C_2(u) \]

Step 4: Compute \( r \)

\[ r = \frac{\partial^2 C_2}{\partial u^2} \left[ \nabla u^T \cdot D \cdot \nabla u \right] \]

Transport

\[ \frac{\partial^2 C_2}{\partial u^2} = \frac{2K}{(u^2 + 4K)^{3/2}} \]

Chemistry
Solution of binary system for pulse input

\[ u(x, t) = u_0 + \frac{18}{(2\pi)^{1/2}} \frac{u_e}{\bar{V}_d} \exp \left( -\frac{1}{2} \left( \frac{(x - Vt)^2}{2tD_L} + \frac{y^2 + z^2}{2tD_T} \right) \right) \]

\[ \bar{V}_d = 55 \]
\[ u_0 / \sqrt{K} = -20 \]
\[ u_e / \sqrt{K} = -30 \]
Problem 2: mixing of two waters

- Water 1 in proportion $\alpha$; water 2, $1-\alpha$ ($\alpha$ is a conservative quantity)
- Mixing takes place BECAUSE of dispersion: NO DISPERSION, NO MIXING!!!
Problem 2: mixing of two waters (ii)

- With a geochemical problem involving 8 species (6 aq., 2 ct activity)

  SOLUTION: Solving for mixing proportions based e.g. on Cl content (freshwater – saline water problem). Solve ONE LINEAR PDE

- The full approach would be solving 6 coupled PDE’s simultaneously with 4 non-linear identities (huge supercomputer)

- SOLUTIÓN: Solving for mixing proportions based e.g. on Cl content (freshwater – saline water problem). Solve ONE LINEAR PDE

- Then do speciation, which in this case is solving a “simple” 6th order polynomial (pocket calculator/Spread sheet).
Problem 3: NOW UPSCALING

Why heterogeneous?
Well, nature IS HETEROGENEOUS!
\[ D_L(z) = \alpha_L K(z) J / \phi \]

Flow Direction

\[ q_x(z) = K(z) J / \phi \]

\[ q_y = q_z = 0 \]
**ANY HOPE FOR UPSCALING?**

Heterogeneous medium:

\[ u^2 + 4K_e \approx u_0^2 + 4K_e \]

Homogenized medium:

\[ R(t) \approx \Delta u_0^2 \frac{K_e}{4(u_0^2 + 4K_e)^{3/2}} \left( 2\alpha_L \frac{J}{\phi} \right)^{-1/2} \exp \left( -\frac{\mu}{2} + \frac{\sigma^2}{8} \right) t^{-3/2} \]

\[ R(t) \approx \Delta u_0^2 \frac{K_e}{4(u_0^2 + 4K_e)^{3/2}} \left( 2\alpha_L \frac{J}{\phi} \right)^{-1/2} \exp \left( -\frac{\mu}{2} - \frac{\sigma^2}{4} \right) t^{-3/2} \]

Discrepancy !!!

**Discrepancy !!!**

\[ K_{\text{eff}} = K_A = \exp \left( \mu + \frac{\sigma^2}{2} \right) \]
Heterogeneous Solution
(stratified system)

Log-normal K distribution

Mean Reaction Rates

$D_T = 0$

$t = 0.1$ days $\sigma^2_y = 1.0$
The experiment (Gramling et al., ES&T, 2002)

1-D sand-filled column, constant EDTA$^4-$ initial concentration, flow from left to right (controlled), CuSO$_4$ injected continuously at the left boundary.

Homogeneous irreversible reaction

At the time-scale of the experiment, reaction can be considered instantaneous: $A + B \rightarrow C$  (meaning A, B cannot coexist at the same point)
1st modeling approach: ADE + Instantaneous equilibrium

ADE + instantaneous equilibrium does not reproduce the observations properly:

(a) It predicts a normalized concentration peak \((C/C_0)\) which DOES NOT CHANGE with time

(b) It over-predicts the total produced mass
Key idea: incomplete mixing can be modeled at a continuum-scale by adding a kinetic (non-instantaneous) reactive term to the driving equation

Species $A$ does not completely and instantaneously react with an already existing species $B$, as part of the pore water is not instantaneously accessible. This results in slow and incomplete mixing at a macroscale.

\[
\frac{\partial c_i}{\partial t} = -v \frac{\partial c_i}{\partial x} + D \frac{\partial^2 c_i}{\partial x^2} - \beta c_A c_B ; \quad i = A, B
\]
Soil: 120g of d < 1mm (from Test Site)

WATER: 240mL
(SYNTHETIC, similar to recharge water at Test Site)

Headspace: 10mL

Additional e⁻ acceptor:
O₂ or NO₃ or Mn oxides or Fe oxides or SO₄²⁻

Additional e⁻ donor:
Na-acetate & MeOH as easily degradable organic substrate

Suite of batch experiments under controlled redox conditions

But we should also worry about organic chemistry
Extremely relevant for water quality purposes
Need Probabilistic Concepts!
1) Identifying contaminant source releases & environmentally sensitive targets.

2) Data acquisition used to infer modeling parameters! Site characterization.

3) Final task: Estimate human health risk toward decision making! Should a site be remediated or not? Is the exposed population at risk?
System Failure

Critical Concentrations

Sources-Receptors

Pathways-Processes
Final remarks

- As engineers we face a daily challenge of solving complex problems, a lot of information is needed.
- As scientists/researchers we deal with the advancement of science, uncertainty should be treated in a rigorous manner, starting at the small scale, then upscaling, then providing estimates in terms of probabilities.
- As professionals living in a real world we need to get the best of both worlds (Hanna Montana dixit). We have to make an effort to convey our clients the problems associated to our job and the impossibility to produce exact results when data is never extensive.
- More research; i.e., more money is needed!
In memoriam

Gerhard H. Jirka
1944 – 2010
Arthur T. Ippen Award recipient (1989)